

## Chapter 10 Permutations & Combinations

0606/22/F/M/19

1. A band can play 25 different pieces of music. From these pieces of music, 8 are to be selected for a concert.
- a. Find the number of different ways this can be done.

$${}^{25}C_8 = 1081575 \quad [1]$$

The 8 pieces of music are then arranged in order.

- b. Find the number of different arrangements possible.

$${}_8P_8 = 40320 \quad [1]$$

The band has 15 members. Three members are chosen at random to be the treasurer, secretary and agent.

- c. Find the number of ways in which this can be done.

$${}_{15}P_3 = 2730 \quad [1]$$

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2. (a) Eight books are to be arranged on a shelf. There are 4 mathematics books, 3 geography books and 1 French book.

(i) Find the number of different arrangements of the books if there are no restrictions.

$$8! = 40320$$

[1]

(ii) Find the number of different arrangements if the mathematics books have to be kept together.

M G G G F      $5! \times 4! = 2880$

[3]

(iii) Find the number of different arrangements if the mathematics books have to be kept together and the geography books have to be kept together.

M G F      $3! \times 4! \times 3! = 864$

[3]

(b) A team of 6 players is to be chosen from 8 men and 4 women. Find the number of different ways this can be done if

(i) there are no restrictions,

$${}^{12}C_6 = 924$$

[1]

(ii) there is at least one woman in the team

[2]

$$924 - \text{no woman}$$

$$924 - {}^8C_6 = 896$$

${}^4W$	${}^8M$	
4	2	$\rightarrow {}^4C_4 \times {}^8C_2$
3	3	$\rightarrow {}^4C_3 \times {}^8C_3$
2	4	$\rightarrow {}^4C_2 \times {}^8C_4$
1	5	$\rightarrow {}^4C_1 \times {}^8C_5$
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		896

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3. (a) Jack has won 7 trophies for sport and wants to arrange them on a shelf. He has 2 trophies for cricket, 4 trophies for football and 1 trophy for swimming. Find the number of different arrangements if

(i) there are no restrictions,

$$7! = 5040$$

[1]

(ii) the football trophies are to be kept together,

F C C S



$$4! \times 4! = 576$$

[3]

(iii) the football trophies are to be kept together and the cricket trophies are to be kept together.

F C S  $\Rightarrow 3! \times 4! \times 2! = 288$

[3]

(b) A team of 8 players is to be chosen from 6 girls and 8 boys. Find the number of different ways the team may be chosen if

(i) there are no restrictions,

$${}^{14}C_8 = 3003$$

[1]

(ii) all the girls are in the team,

$${}^6C_6 \times {}^8C_2 = 28$$

[1]

(iii) at least 1 girl is in the team.

$$3003 - {}^8C_8 = 3002$$

[2]

4.(a) Eleven different television sets are to be displayed in a line in a large shop.

(i) Find the number of different ways the televisions can be arranged.

$$11! = 39916800 \quad [1]$$

Of these television sets, 6 are made by company A and 5 are made by company B.

(ii) Find the number of different ways the televisions can be arranged so that no two sets made by company A are next to each other.

$$A B A B A B \quad {}^6P_6 \times {}^5P_5 = 86400 \quad [2]$$

(b) A group of people is to be selected from 5 women and 3 men.

(i) Calculate the number of different groups of 4 people that have exactly 3 women.

$${}^5C_3 \times {}^3C_1 = 30 \quad [2]$$

(ii) Calculate the number of different groups of at most 4 people where the number of women is the same as the number of men.

$$\begin{array}{l} W \\ 2 \\ 1 \end{array} \quad \begin{array}{l} M \\ 2 \\ 1 \end{array} \rightarrow \begin{array}{l} {}^5C_2 \times {}^3C_2 \\ {}^5C_1 \times {}^3C_1 \end{array} \quad [2]$$


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$$45$$

5.(a) Jess wants to arrange 9 different books on a shelf. There are 4 mathematics books, 3 physics books and 2 chemistry books. Find the number of different possible arrangements of the books if

(i) there are no restrictions,

$$9! = 362880 \quad [1]$$

(ii) a chemistry book is at each end of the shelf,

$$\begin{array}{c} \underline{C} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \underline{C} \\ \underbrace{\hspace{10em}} \\ 7! \times 2 = 10080 \end{array} \quad [2]$$

(iii) all the mathematics books are kept together and all the physics books are kept together.

$$\begin{array}{c} \underline{M} \quad \underline{P} \quad \underline{C} \quad \underline{C} \\ \begin{array}{l} \text{red arrow from } \underline{M} \text{ to } 4! \\ \text{blue arrow from } \underline{P} \text{ to } 3! \end{array} \\ 4! \times 4! \times 3! = 3456 \end{array} \quad [3]$$

(b) A quiz team of 6 children is to be chosen from a class of 8 boys and 10 girls. Find the number of ways of choosing the team if

(i) there are no restrictions,

$${}^{18}C_6 = 18564$$

[1]

(ii) there are more boys than girls in the team

$$\begin{array}{l} 8 \\ \text{B} \\ 6 \\ 5 \\ 4 \end{array} \quad \begin{array}{l} 10 \\ \text{G} \\ 0 \\ 1 \\ 2 \end{array} \quad \begin{array}{l} {}^8C_6 \\ {}^8C_5 \times {}^{10}C_1 \\ {}^8C_4 \times {}^{10}C_2 \end{array}$$

[4]

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$$\begin{aligned} &28 + 560 + 3150 \\ &= 3738 \end{aligned}$$



6. (a) A 5-digit code is to be chosen from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit may be used once only in any 5-digit code. Find the number of different 5-digit codes that may be chosen if

(i) there are no restrictions,

$${}^9P_5 = 15120$$

[1]

(ii) the code is divisible by 5,

$$\text{--- -- } \underline{5} \quad {}^8P_4 = 1680$$

[1]

(iii) the code is even and greater than 70 000.

$$\begin{array}{l} \underline{7} \text{ ---} \\ \underline{8} \\ \underline{9} \end{array} \quad \begin{array}{l} \underline{2} \text{ or } \underline{4} \text{ or } \underline{6} \text{ or } \underline{8} \\ \underline{1}, \underline{3}, \underline{5} \\ \underline{2}, \underline{4}, \underline{6}, \underline{8} \end{array}$$

$$\begin{array}{l} \underline{7} \text{ ---} \underline{2} \\ \hookrightarrow {}^7P_3 \times 11 \\ = 2310 \end{array}$$

[3]

(b) A team of 6 people is to be chosen from 8 men and 6 women. Find the number of different teams that may be chosen if

(i) there are no restrictions,

$${}^{14}C_6 = 3003$$

[1]

(ii) there are no women in the team,

$${}^8C_6 = 28$$

[1]

(iii) there are a husband and wife who must not be separated.

$$\begin{array}{l} \text{both} \\ \text{in} \\ {}^{12}C_4 \\ 495 \end{array} + \begin{array}{l} \text{both} \\ \text{out} \\ {}^{12}C_6 \\ 924 \end{array} = 1419$$

[3]

7. A 5-digit code is formed using the following characters.

✓ **5** Letters    a   e   i   o   u

✓ **6** Numbers    1   2   3   4   5   6

✓ **3** Symbols    @   \*   #

No character can be repeated in a code. Find the number of possible codes if

(i) there are no restrictions,

$${}^{14}P_5 = 240240$$

[2]

(ii) the code starts with a symbol followed by two letters and then two numbers,

$$\begin{array}{cccc} \underline{S} & \underline{L} & \underline{L} & \underline{N} & \underline{N} \\ | & & & & \\ 3P_1 & \times & 5P_2 & \times & 6P_2 = 1800 \end{array}$$

[2]

(iii) the first two characters are numbers, and no other numbers appear in the code.

$$\begin{array}{cccc} \underline{N} & \underline{N} & \underline{\quad} & \underline{\quad} & \underline{\quad} \\ & & \underbrace{\hspace{2cm}} & & \\ & & \text{Not number} & & \end{array}$$

↙

$$6P_2 \times 8P_3 = 10080$$

[2]